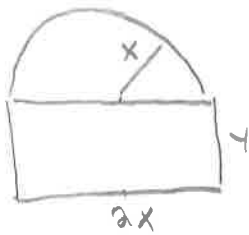


Optimization Problems

①



$$2 + \pi = 4x + 2y + \pi x$$

$$2y = 2 + \pi - 4x - \pi x$$

$$y = \frac{2 + \pi - 4x - \pi x}{2}$$

$$A = 2xy + \frac{1}{2}\pi x^2$$

$$A(x) = x(2 + \pi - 4x - \pi x) + \frac{1}{2}\pi x^2$$
$$= 2x + \pi x - 4x^2 - \pi x^2 + \frac{1}{2}\pi x^2$$

$$\frac{dA}{dx} = 2 + \pi - 8x - \pi x$$

$$2x + \pi x = 2 + \pi$$

$$x(2 + \pi) = 2 + \pi$$

$$x = 1$$

Rectangle: 2 x 1

②



$$\pi r^2 h = 1$$

$$h = \frac{1}{\pi r^2}$$

$$C = 2\pi r^2 \cdot 1 + 2\pi r h \cdot 18$$

$$C(r) = 18\pi r^2 + 36\pi r \left(\frac{1}{\pi r^2}\right)$$

$$\frac{dC}{dr} = 36\pi r - 36r^{-2}$$

$$\frac{dC}{dr} = 0, \quad 36\pi r = \frac{36}{r^2}$$

$$r^3 = \frac{1}{\pi}$$

$$r = \sqrt[3]{\frac{1}{\pi}}$$

$$h = \sqrt[3]{\frac{1}{2\pi}}$$

③



$$2x + 2y = 36$$

$$x + y = 18$$

$$x = 2\pi r$$

$$r = \frac{x}{2\pi}$$

$$y = 18 - x$$

$$V = \pi r^2 y$$

$$V(x) = \pi \left(\frac{x}{2\pi}\right)^2 (18 - x)$$

$$V = \frac{x^2(18-x)}{4\pi} = \frac{1}{4\pi}(18x^2 - x^3)$$

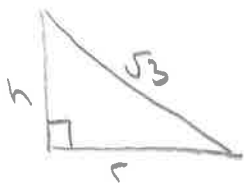
$$\frac{dV}{dx} = \frac{1}{4\pi}(36x - 3x^2)$$

$$\frac{dV}{dx} = 0, \quad 36x - 3x^2 = 0$$

$$x = 0, \quad x = 12$$

$$y = 6$$

④



$$h^2 + r^2 = 3$$

$$r^2 = 3 - h^2$$

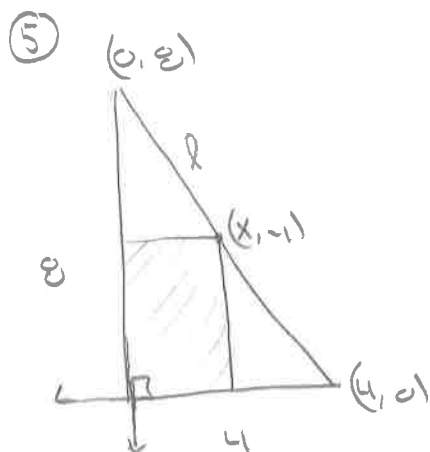
$$V = \frac{1}{3}\pi r^2 h$$

$$V(h) = \frac{1}{3}\pi h(3 - h^2)$$

$$V(h) = \pi h - \frac{1}{3}\pi h^3$$

$$\frac{dV}{dh} = \pi - \pi h^2 = \pi(1 - h^2)$$

$$\frac{dV}{dh} = 0, \quad h = 1, \quad r = \sqrt{2}, \quad V = \frac{2}{3}\pi$$



the line l , $y = -2x + 8$

$$V = \pi x^2 y$$

$$V(x) = \pi x^2 (-2x + 8)$$

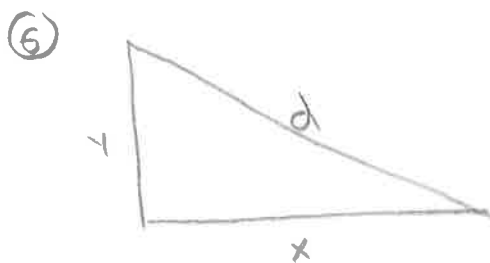
$$= -2\pi x^3 + 8\pi x^2$$

$$\frac{dV}{dx} = -6\pi x^2 + 16\pi x$$

$$\frac{dV}{dx} = 0, \quad 6\pi x^2 = 16\pi x$$

$$x = \frac{8}{3}, \quad y = -2\left(\frac{8}{3}\right) + 8 = \frac{8}{3}$$

$$V = \frac{512\pi}{27}$$



$$d = \sqrt{x^2 + y^2} \quad x + y = 8, \quad y = 8 - x$$

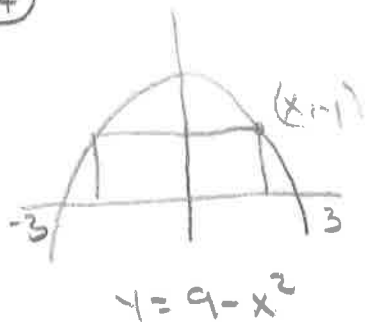
$$d(x) = \sqrt{x^2 + (8 - x)^2} = (x^2 + 64 - 16x + x^2)^{1/2}$$

$$\frac{dd}{dx} = \frac{1}{2} (2x^2 - 16x + 64)^{-1/2} (4x - 16)$$

$$\frac{dd}{dx} = \frac{4x - 16}{2\sqrt{2x^2 - 16x + 64}}$$

$$\frac{dd}{dx} = 0, \quad x = 4, \quad y = 4, \quad d = \sqrt{32}$$

⑦



$$A = 2x \cdot y$$

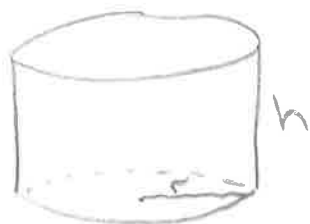
$$A(x) = 2x(9 - x^2)$$

$$A(x) = 18x - 2x^3$$

$$\frac{dA}{dx} = 18 - 6x^2$$

$$\frac{dA}{dx} = 0, \quad x = \sqrt{3}, \quad y = 6$$

⑧



$$V = \pi r^2 h$$

$$300\pi = \pi r^2 h$$

$$h = \frac{300}{r^2}$$

$$C = 2 \cdot 2\pi r^2 + 6 \cdot 2\pi r h$$

$$C(r) = 4\pi r^2 + \frac{3600\pi}{r}$$

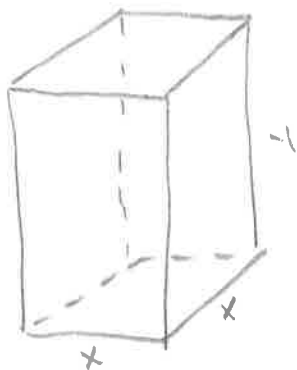
$$\frac{dC}{dr} = 8\pi r - \frac{3600\pi}{r^2}$$

$$\frac{dC}{dr} = 0, \quad 8\pi r = \frac{3600\pi}{r^2}$$

$$r^3 = \frac{3600\pi}{8\pi} = 450$$

$$r = \sqrt[3]{450}, \quad h = \frac{300}{(450)^{2/3}}$$

9



$$V = x^2 y$$

$$300 = x^2 y$$

$$y = \frac{300}{x^2}$$

$$C = 2 \cdot 2x^2 + 6 \cdot 4xy$$

$$C(x) = 4x^2 + 24x \left(\frac{300}{x^2} \right)$$

$$C(x) = 4x^2 + \frac{7200}{x}$$

$$\frac{dC}{dx} = 8x - \frac{7200}{x^2}$$

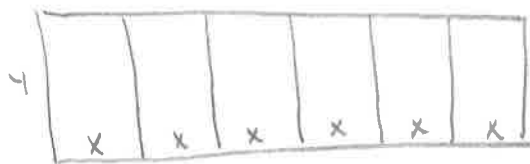
$$\frac{dC}{dx} = 0, \quad 8x = \frac{7200}{x^2}$$

$$x^3 = 900$$

$$x = \sqrt[3]{900}$$

$$y = \frac{300}{(900)^{2/3}}$$

10



$$6xy = 60$$

$$xy = 10$$

$$y = \frac{10}{x}$$

$$C = 10 \cdot 12x + 10 \cdot 2y + 5 \cdot 5y$$

$$C = 120x + 45y$$

$$C(x) = 120x + 45 \left(\frac{10}{x} \right)$$

$$= 120x + \frac{450}{x}$$

$$\frac{dC}{dx} = 120 - \frac{450}{x^2}$$

$$\frac{dC}{dx} = 0, \quad x^2 = \frac{450}{120} = \frac{15}{4}$$

$$x = \frac{\sqrt{15}}{2}, \quad y = \frac{20}{\sqrt{15}}$$

$$C = \$464.76$$

